International Journal of Engineering Research & Management Technology ISSN: 2348-4039
January-2016 Volume 3, Issue-1
Email: editor@ijermt.org www.ijermt.org

Study of Inventory Model for Decaying Items with Stock Dependent Demand, Time Importance of Money above Acceptable Delay in Payments

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ABSTRACT:
In this paper, we have taken a more realistic demand rate that depends on two factors, one is time, and the second is the stock level available. The stock level in itself obviously gets depleted due to the customer's demand. As a result, what we witness here is a circle in which the customer's demand is being influenced by the level of stocks available, while the stock levels are getting depleted due to the customer's demands. This assumption takes the customer's interests as well as the market forces into account. The demand rate is such that as the inventory level increases, it helps to increase the demand for the inventory under consideration. While as the time passes, demand is depends upon the various factors. The competitive nature of the market has been accounted for by taking permissible delay in payments into consideration. Finally, the results have been illustrated with the help of numerical examples.

KEY WORDS: Customer's demand, stock level.

1. INTRODUCTION:

The classical deterministic inventory models consider the demand rate to be either constant or time-dependent but independent of the stock status. However, for certain types of consumer goods of inventory, the demand rate may be influenced by the stock level. It has been noted by marketing researchers and practitioners that an increase in a product's shelf space usually has a positive impact on the sales of that product and it is usually observed that a large pile of goods on shelf in a supermarket will lead the customer to buy more, this occurs because of its visibility, popularity or variety and then generate higher demand. In such a case, the demand rate is no longer a constant, but it depends on the stock level. This phenomenon is termed as 'stock dependent consumption rate'. In general, 'stock dependent consumption rate' consists of two kinds. One is that the consumption rate is a function of order quantity and the other is that the consumption rate is a function of inventory level at any instant of time.
The consumption rate may go up or down with the on-hand stock level. These phenomena attract many marketing researchers to investigate inventory models related to stock-level. Conversely, low stocks of certain baked goods (e.g., donuts) might raise the perception that they are not fresh. Therefore, demand is often time and inventory-level dependent.

2. REVIEW OF LITERATURE:

At present, great interest has been shown in developing mathematical models in the presence of trade credits. Kingsman (1983), Chapman et al. (1985) and Daellenbach (1986) have developed the effect of the trade credits on the optimal inventory policy. Trade credit has been a topic of interest for many authors in inventory policy like Hayley and Higgings (1973), Davis and Gaither (1985), Ouyang et al. (2004), Ward and Chapman et al. (1988). Recently, Khanna et al. (2011) developed an EOQ model for deteriorating items with time dependent demand under permissible delay in payments. Inventory models with permissible delay in payments were first studied by Goyal (1985). Shinn et al. (1996) extended Goyal's (1985) model by considering quantity discounts for freight cost. Chu et al. (1998) and Chung et al. (2001) also extended Goyal's model for the case of deteriorating items. Sana and Chaudhury (2008) developed a more general EOQ model with delay in payments, price discounting effect and different types of demand rate. All the above articles are based on the assumption that the cost is constant over the planning horizon. This assumption may not be true in real life, as many countries have high inflation rate. Inflation also influences demand of certain products. As inflation increases, the value of money goes down. As a result, while determining the optimal inventory policy, the effect of inflation and time value of money cannot be ignored. Buzacott (1975) discussed EOQ model with inflation subject to different types of pricing policies. Wee and Law (1999) established the problem with finite replenishment rate of deteriorating items taking account of time value of money. Chang (2004) developed an inventory model for deteriorating items under inflation under a situation in which the supplier provides the purchaser with a permissible delay of payments if the purchaser orders a large quantity. Recently, Tripathi (2011) developed an inventory model under which the supplier provides the purchaser a permissible delay in payments if the purchaser orders a large quantity.

This paper is the extension of Tripathi (2011) paper in which demand rate is time dependent and deterioration rate is zero. Liao (2007) established the inventory replenishment policy for deteriorating items in which the supplier provides a permissible delay in payments if the purchaser orders a large quantity. Hon and Lin (2009) developed an inventory model to determine an optimal ordering policy for deteriorating items with delayed payments permitted by the supplier under inflation and time discounting. Teng et al. (2012) proposed an EOQ model in which the constant demand to a linear, non- decreasing demand functions of time, which is suitable not only for the growth stage but also for the maturity stage of the product life cycle. Khanra et al. (2011) developed an EOQ model for deteriorating item having time dependent developed an EOQ model for deteriorating item having time- dependent demand when delay in payment is permissible. Yang et al. (2010) developed an EOQ model for deteriorating items with stock- dependent demand and partial backlogging. Ouyang and Chang (2013) extended the effects of the reworking imperfect quality item and trade credit on the EPQ model with imperfect production process and complete backlogging. Soni (2013) developed an EOQ model considering (i) the demand rate as multivariate function of price and level of inventory (ii) delay in payment is permissible. Taleizadeh and Nematollahi (2014) investigated the effect of time- value of money and inflation on the optimal ordering policy in an inventory control system. Sarkar et al. (2014) developed an economic manufacturing quantity (EMQ) model for the selling price and the time dependent demand pattern in an imperfect production process. Teng et al. (2013) developed an EOQ model extending the constant demand to a linear, non- decreasing demand function of time and incorporate a permissible delay in payment in payment under two levels of trade credit into the model. Tripathi and Pandey (2013) presented an inventory model for deteriorating items with Weibull distribution time dependent demand rate under permissible delay in payments. Sarkar (2012) presented an EOQ model for finite replenishment rate where demand and deterioration rate are both time dependent. Tripathi (2011) established an inventory model for non- deteriorating item and time dependent demand rate under inflation when the supplier offers a permissible delay to the purchaser, if the order quantity is greater than or equal to a predetermined quantity. Some articles related to the inventory policy under delay in payments can be found in Mirzazadeh and Moghaddam (2013), Mirzazadah et al. (2009), Teheri et al. (2013) and their references.

3 ASSUMPTIONS AND NOTATIONS:

The mathematical models of the two warehouse inventory problems are based on the following assumptions and notations:

ASSUMPTIONS:

1. The inventory system involves a single type of items.
2. Demand rate is dependent on time and stock level.
3. Deterioration rate is taken as Kt.
4. Shortages are not permitted.
5. The replenishment rate is instantaneous.

NOTATIONS:

1. $D = a + bt + cI(t)$ Time and Stock dependent demand
2. C_O = Ordering cost
3. C_h = holding cost per unit time, excluding interest charges
4. C_P = purchasing cost which depends on the delay period and supplier's offers
5. p = selling price per unit
6. M = permissible delay period
7. M_i = i^{th} permissible delay period in settling the amount
8. $_i$ = discount rate (in %) of purchasing cost at i-th permissible delay period.
9. i_e = rate of interest which can be gained due to credit balance
10. i_c = rate of interest charged for financing inventory
11. T = length of replenishment
12. $AP_1(T, M_i)$ = average profit of the system for $T \geq M_i$
13. $AP_2(T, M_i)$ = average profit of the system for $T \leq M_i$
14. Q_0 = Initial lot size

MODEL FORMULATION AND SOLUTION:

The cycle starts with initial lot size Q_0 and ends with zero inventory at time t=T. Then the differential equation governing the transition of the system is given by

$$\frac{dI(t)}{dt} = -Kt - (a + bt + cI(t)), \qquad 0 \leq t \leq T \qquad \text{... (1.1)}$$

With boundary condition I(0) = Q_0

The purchasing cost at different delay periods are

$$C_P = \begin{cases} C_r(1-\delta_1), M = M_1 \\ C_r(1-\delta_2), M = M_2 \\ C_r(1-\delta_3), M = M_3 \\ \infty, M > M_3 \end{cases}$$

Where C_r = maximum retail price per unit.

And M_i (i=1,2,3) = decision point in settling the account to the supplier at which supplier offers δ% discount to the retailer.

Now two cases may occur:

1. When $T \geq M$
2. When $T < M$

Case 1: when $T \geq M$

Solving the equation (1), we get

$$\frac{dI(t)}{dt} + KtI(t) = -(a + bt + cI(t))$$

Using the boundary condition I(0) = Q_0, we get

c = Q_0

Therefore the solution of equation (1) is

$$I(t) = \left\{ Q_0 - at - \frac{(a+b)}{2}t^2 - \left(\frac{aK}{2} + b\right)\frac{t^3}{3} - \frac{bK}{8}t^4 \right\} e^{-t-Kt^2/2} \qquad 0 \leq t \leq T \qquad \text{... (1.2)}$$

In this case it is assumed that that the replenishment cycle T is larger than the credit period M.

The holding cost, excluding interest charges is

$$HC = C_h \int_0^T I(t) e^{-rt} dt$$

$$\begin{aligned}HC = C_h \Bigg[& \left\{ Q_0 T - \frac{a}{2} T^2 - \frac{(a+b)}{6} T^3 - \left(\frac{aK}{2} + b \right) \frac{T^4}{12} - \frac{bK}{40} T^5 \right\} \\ & -(1+r) \left\{ \frac{Q_0}{2} T^2 - \frac{a}{3} T^3 - \frac{(a+b)}{8} T^4 - \left(\frac{aK}{2} + b \right) \frac{T^5}{15} - \frac{bK}{48} T^6 \right\} \\ & - \frac{K}{2} \left\{ \frac{Q_0}{3} T^3 - \frac{a}{4} T^4 - \frac{(a+b)}{10} T^5 - \left(\frac{aK}{2} + b \right) \frac{T^6}{18} - \frac{bK}{56} T^7 \right\} \Bigg] \end{aligned} \quad \text{... (1.3)}$$

The cost of financing inventory during time span [M,T] is

$$FC = i_c C_P \int_M^T I(t) e^{-r(M+t)} dt$$

$$\begin{aligned} FC = i_c C_P \Bigg[& (1-rM) \left\{ Q_0 T - \frac{a}{2} T^2 - \frac{(a+b)}{6} T^3 - \left(\frac{aK}{2} + b \right) \frac{T^4}{12} - \frac{bK}{40} T^5 \right\} \\ & -(1+r) \left\{ \frac{Q_0}{2} T^2 - \frac{a}{3} T^3 - \frac{(a+b)}{8} T^4 - \left(\frac{aK}{2} + b \right) \frac{T^5}{15} - \frac{bK}{48} T^6 \right\} \\ & - \frac{K}{2} \left\{ \frac{Q_0}{3} T^3 - \frac{a}{4} T^4 - \frac{(a+b)}{10} T^5 - \left(\frac{aK}{2} + b \right) \frac{T^6}{18} - \frac{bK}{56} T^7 \right\} \\ & -(1-rM) \left\{ Q_0 M - \frac{a}{2} M^2 - \frac{(a+b)}{6} M^3 - \left(\frac{aK}{2} + b \right) \frac{M^4}{12} - \frac{bK}{40} M^5 \right\}. \\ & +(1+r) \left\{ \frac{Q_0}{2} M^2 - \frac{a}{3} M^3 - \frac{(a+b)}{8} M^4 - \left(\frac{aK}{2} + b \right) \frac{M^5}{15} - \frac{bK}{48} M^6 \right\} \\ & + \frac{K}{2} \left\{ \frac{Q_0}{3} M^3 - \frac{a}{4} M^4 - \frac{(a+b)}{10} M^5 - \left(\frac{aK}{2} + b \right) \frac{M^6}{18} - \frac{bK}{56} M^7 \right\} \Bigg] \end{aligned} \quad \text{... (1.4)}$$

Opportunity gain due to credit balance during time span [0,M] is

$$Opp.Gain = i_e p \int_0^M (M-t)(a + bt + cI(t)) e^{-rt} dt$$

$$Opp.Cost = i_e p \left[(a+bM) \frac{e^{-rM}}{r^2} - 2b \frac{e^{-rM}}{r^3} + \frac{aM}{r} - \frac{(a-bM)}{r^2} + \frac{2b}{r^3} \right] \quad \text{... (1.5)}$$

Therefore, the total cost is given by

TC_{1i}=Purchasing Cost +holding cost +ordering cost +interest charged-interest earned for $M \in \{M_1, M_2, M_3\}$

$$TAC_{1i} = \frac{1}{T} TC_{1i} \quad \text{... (1.6)}$$

Case 2 when T < M

In this case, credit period is larger than the replenishment cycle consequently cost of financing inventory is zero. The holding cost, excluding interest charges is

$$HC = C_h \int_0^T I(t) e^{-rt} dt$$

$$HC = C_h \left[\left\{ Q_0 T - \left(\frac{a}{2} T^2 + \frac{b}{6} T^3 + \frac{aK}{24} T^4 + \frac{bK}{40} T^5 \right) \right\} \right.$$

$$-r\left\{\frac{Q_0}{2}T^2-\left(\frac{a}{3}T^3+\frac{b}{8}T^4+\frac{aK}{30}T^5+\frac{bK}{48}T^6\right)\right\}$$
$$-\frac{K}{2}\left\{\frac{Q_0}{3}T^3-\left(\frac{a}{4}T^4+\frac{b}{10}T^5+\frac{aK}{36}T^6+\frac{bK}{56}T^7\right)\right\}\Bigg] \quad ...(1.7)$$

Opportunity gain due to credit balance during time span [0,M] is

$$Opp.Gain = i_e p\left[\int_0^T (T-t)(a+bt)e^{-rt}dt+\int_0^T (M-T)(a+bt)e^{-rt}dt\right]$$
$$= i_e p\left[\int_0^T \{aT+(bT-a)t-bt^2\}e^{-rt}dt+(M-T)\int_0^T \{a+bt\}e^{-rt}dt\right] \quad ..... (1.8)$$

Therefore the total cost during the time interval T is given by

TC_{2i}=Purchasing cost +holding cost +ordering cost-interest earned (Opp. cost)

$$TAC_{2i}=\frac{1}{T}TC_{2i} \quad ..... (1.9)$$

Now, our aim is to determine the optimal value of T and M such that TAC(T,M) is minimized where

$$TAC(T,M)=Inf.\begin{cases}TAC_{1i}(T,M),TAC_{2i}(T,M)\\ where, M\varepsilon(M_1,M_2,M_3)\end{cases} \quad ..... (1.10)$$

Numerical Example

We consider the value of the parameters as follows

a=100, b=50,
C_0=200/Order, C_h=0.05/unit/month
C_r=125/unit, p=175/unit,

i_c=0.15, i_e=0.12,
M_1=3months, M_2=5months,
M_3=7months, K=0.001
$_1$=20% δ $_2$=15%
$_3$=5%

The optimal solution are given by

$TC_{11}=1.268\times10^5$ T=32months (for M_1)
$TC_{21}=3.1887\times10^5$ T=27months (for M_2)
$TC_{31}=3.5617\times10^5$ T=8months (for M_3)

Table 1.1: Variation in TC with the variation in a

a	T	$TC(10^5)$
70	31.7654	6.5256
80	34.1821	6.1423
90	33.9932	1.7233
100	32.0032	1.2629
110	32.8276	1.8945
120	30.1763	1.3581
130	21.9634	4.5273

Table 1.2: Variation in TC with the variation in b

b	T	$TAC(10^5)$
30	21.235	10.2987
35	28.1254	10.1122
40	30.4457	9.9732

45	33.1896	8.1842
50	33.7832	7.2622
55	34.1457	7.0021
60	34.8485	6.3252
65	31.9517	6.1842
70	36.1725	6.09342
75	38.8954	1.32322

Table 1.3: Variation in TC with the variation in C_h

C_h	T	TAC(10^5)
0.020	18.7241	1.8954
0.025	21.9154	2.3112
0.030	21.1892	2.7776
0.035	27.2431	3.8154
0.040	29.8561	4.1112
0.045	31.1452	4.5772
0.050	32.0002	1.2681
0.055	33.5231	6.3314
0.060	34.1272	6.7821
0.065	31.4139	7.1957

Table 1.4: Variation in TC with the variation in K

K	T	TAC(10^5)
0.0008	32.8081	4.5417
0.0009	32.8081	4.9857
0.0010	32.8081	1.2681
0.0015	32.8081	6.8934
0.0020	32.8081	6.9572
0.0025	32.8081	7.2231
0.0030	32.8081	7.8573
0.0035	32.8081	9.4315
0.0040	32.8081	10.5473
0.0045	32.8081	11.1892

CONCLUSION:

Panda et al. (2009) presented a practical inventory model with pre- and post-deterioration discounts on selling price. The mathematical model is developed in order to investigate how much discount on selling price may be given to maximize the profit per unit time when demand is both stock dependent and constant, but the time value of money and shortage are not considered. In this paper, we developed a model with supplier's trade offer of credit and price discount the purchase of stock. The model considered the both, deterioration effect and time discounting. Generally, supplier offer different price discount on purchase of items of retailer at different delay periods. Suppliers allow maximum delay period, after which they will not take a risk of getting back money from retailers or any other loss of profit. Constant deterioration is not a viable concept; hence, we have considered an inventory with deterioration increasing with time.

REFERENCES:

1. Chang, C.T. (2004). An EOQ model with deteriorating items under inflation when supplier credits linked to order quantity. International Journal of Production Economics, Vol. 88, pp. 307-316.
2. Gholami-Qadikolaei, A., Mirzazadeh, A. & Tavakkoli-Moghaddam, R. (2013). A stochastic multiobjective multiconstraint inventory model under inflationary condition and different inspection scenarios J Engineering Manufacture, Vol.227, No.7, pp. 1057–1074.
3. Khanna,S., Ghosh, S.K. and Chaudhuri, K.S. (2011). An EOQ model for a deteriorating item with time-dependent quadratic demand under permissible delay in payment. Applied Mathematics and Computation, Vol. 218, pp. 1-9.
4. Liao, J.J. (2007). A note on an EOQ model for deteriorating items under supplier credit linked to order quantity. Applied Mathematics Modelling, Vol.31, pp. 1690-1699.
5. Ouyang, L.Y. and Chang, C.T. (2013). Optimal production lot with imperfect production process under permissible delay in payment and complete backlogging. International Journal of Production Economics, Vol. 144, pp. 610-617.
6. Saini A.K. (2012) : On Some Mathematical Models for Decaying Items with Credit Policies, Singhania University, Rajasthan.
7. K.K.Bansal,U.P.Singh,Develop an Inventory Model with Time Dependent Deterioration, and Demand Rate under the Condition of Permissible Delay" International Journal of Engineering Research & Management Technology, vol.1 Issue 5
8. K.K.Bansal,Adarsh Kumar,A Deterministic Inventory Model for a Deteriorating Item Is Explored In an Inflationary Environment for an Infinite Planning Horizon, International Journal of Education and Science Research Review, vol.1 Issue 4
9. K.K.Bansal,Navin Ahlawat, Optimal Ordering Decision for Deteriorated Items with Expiration Date and Uncertain Lead Time International Journal of Management Research
10. Review, Vol.2 Issue 6
11. K.K.Bansal, Order Level Inventory Model With Decreasing Demand And Variable Deterioration, International Journal of Engineering & Science Research, Vol.2 Issue 9
12. Sarkar, B., Mandal, P. and Sarkar, S. (2014). An EMO model with price and time dependent demand under the effect of reliability and inflation. Applied Mathematical and Computation, Vol.231, pp. 414-421.
13. Taheri-Tolgari, J., Mirzazadeh, A., & Jolai, F. (2012). An inventory model for imperfect items under inflationary conditions with considering inspection errors, Computers and Mathematics with Applications, Vol. 63, pp. 1007–1019.
14. Taleizadeh, A. A and Nematollahi, M. (2014). An inventory control problem for deteriorating items with back- ordering and financial consideration. Applied Mathematical Modelling, Vol. 38(1), pp. 93-109.
15. Tripathi, R.P. (2011). Inventory model with time-dependent demand rate under inflation when supplier credit linked to order quantity. International Journal of Business and Information Technology, Vol.1, No.3, 174-183.
16. Yang, H.L., Teng, J.T. and Chern, M.S. (2010). An inventory model under inflation for deteriorating items with stocl-dependent consumption rate and partial backlogging shortages. International Journal of Production Economics, Vol. 123, pp. 8-19.